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THE 28-MONTH PERIOD IN SOLAR ACTIVITY AND CORRESPONDING PERIODS IN MAGNETIC AND METEOROLOGICAL DATA

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SYNOPSIS

Periods in the strict sense of the term have not been discovered either in solar or meteorological data. However, the variation in spottedness of the sun over an irregular period of roughly 11 years represents a type of period which may vary in length systematically, and such periods are found in both classes of data.

The periodogram analysis has given very uncertain indications of the existence of periods of constant length, and it is pointed out that the results of such methods of analysis are entirely consistent with the hypothesis of variability in periods and can be properly interpreted only by the aid of other methods of analysis.

By means of statistical methods results are obtained which, interpreted by certain criteria, indicate the existence and probable length of a more or less regular recurrence. The period is isolated either by employing suitable smoothing formulae or by a free-hand smoothing of graphs or a combination of both methods. Epochs of maxima and minima are then selected from the graphs and their reality determined by non-mathematical criteria which are peculiar to the methods employed.

A summary is given of the various kinds of solar data made use of, and it is pointed out that since only one-half of the solar surface is visible at any time there must necessarily result more or less irregularity in the variations of periods, especially the shorter ones.

The mean heliographic latitude of the entire spotted area has a well-marked 11-year variation in which there is a maximum excess of spots in the northern hemisphere three to four years before the epoch of sunspot maximum and a maximum excess in the southern hemisphere about a year before the sunspot minimum. When the average latitude of spots is high, there is an excess of spots in the northern hemisphere; when low, there is an excess of spots in the southern hemisphere.

When the 11-year variation is eliminated from the latitude data in half-yearly means, there are disclosed short period variations of irregular length, the mode or most frequent length being about 2.4 years. If these variations were of a purely accidental nature, the most frequent length would average about 1.6 years. Self-correlation of the data likewise indicates a well-marked recurrent feature of about 2¼ years in length. These two results should be regarded, in view of the strictly impersonal character of the results and the wide divergence from random data, as adequate evidence for the existence of a periodicity in the latitude data. Definitive epochs of maxima and minima from 1855 to 1925 are given and the average length of the period is found to be 2.55 years with the most frequent interval 2.50 years.

After the 11-year variation is eliminated, this short period appears in the relative numbers as well as in their scatter, as measured by the interdiurnal variability. It is less regularly shown, however, than in the latitude data.

The epochs of maxima and minima derived from the relative numbers indicate that the length of the period is around two years at sunspot maxima and three years at sunspot minima. This result is consistent with the fact that a relatively short interval occurs between a minimum and a maximum phase when the latter is exceptionally intense.

There are given new determinations of the epochs of maxima and minima of the elemental 11-year variation in the relative numbers.

New determinations of the 11-year epochs of maximum and minimum magnetic declination range are given and there are derived epochs of the 28-month variation in the range. These epochs average about .07 year later than the corresponding epochs for the relative numbers.

An 11-year variation occurs in the length of the 28-month period in terrestrial temperatures with long and short intervals about four years after the Wolfer epochs of minima and maxima respectively.

The short period variations in temperature have a closer causal relation to the solar latitude than to the relative number variations. It is found from data extending over a period of 75 years that the epochs of maximum and minimum temperature occur about one year later than the epochs of maximum southern and northern heliographic latitude respectively, and about four months before the epochs of minimum and maximum spottedness.

A discussion of the significance and evidential value of the results of the investigation follows and certain criticisms are answered.

INTRODUCTION

The dependence of the meteorological phenomena of the earth upon the variations of solar activity has been affirmed by many investigators. Two important relations seem to be well established, viz, the inverse relation between both the temperature and the pressure in the equatorial regions and the solar spottedness. The relation between temperature and spottedness holds also for the entire globe but the amplitude of the variation decreases with increasing distance from the tropics. Employing yearly means of the meteorological elements and of the sunspot relative numbers, an 11-year variation in temperature is found with phases in opposition to those of the solar cycle.

A variation in the number of prominences at intervals averaging three to four years has been compared with meteorological elements with some success by Lockyer and Bigelow, but the results have been rather more suggestive than conclusive. A three-year variation in temperature and pressure in the equatorial regions, particularly in the Indian Ocean area, has been studied by numerous investigators. The correlation of monthly values of meteorological elements and the sunspot numbers has not led to any decisive results.

Practically all investigators of solar and meteorological variations have assumed that periods should be of uniform length and accordingly have employed the periodogram analysis, but the results have not favored the hypothesis of the existence of such periods, and the reasonable inference is that they do not exist in meteorological data, since they have not been found in solar data. However, it must not be lost sight of that the results of such a periodogram analysis are not inconsistent with the possibility of the existence of periods variable within reasonable limits and rightly interpreted can not conflict with the findings by other methods.

On the other hand, all available evidence, properly interpreted, points to the existence of periods with a certain average length but variable within limits such that the longest period may be twice the shortest. The so-called 11-year period of solar spots is, it is well known, not of uniform length. The best value of the period as determined by Newcomb employing least square methods is 11.13 years. During the last 300 years the 56 intervals,

maximum to maximum and minimum to minimum, have ranged between 7.3 and 17.1 years, half of them being less than 10 or greater than 12 years. The writer has shown elsewhere (*1*) that this variation, instead of being accidental, is in reality highly systematic, being controlled by a 36-year period, so that it is possible to predict the next epoch of maximum or minimum with a higher degree of accuracy, in the long run, than if merely the normal interval of 11.13 years had been employed. As to the short meteorological period of about three years, referred to above, the writer in a recent paper (**2**) showed that it is clearly apparent in high latitudes of the Northern Hemisphere as well as in the Tropics. Its length averages 28 months but varies systematically under control of the solar 11-year and 36-year periods. He further showed that this period exists in solar phenomena.

The propriety of designating by the term "period" a sequence which varies systematically is questioned by some meteorologists. However, astronomers have long been accustomed to designate as periods such irregular recurrences as that of maximum solar spottedness and the maximum brilliancy of variable stars. We may, therefore, regard this practice as an instance of a meaning inconsistent with the primary significance of a word but sanctioned by usage.

The present investigation has for its object the determination of definitive epochs of maxima and minima for the 11-year and 28-month periods in various solar and magnetic phenomena. From these epochs we derive the average length of the periods as well as the systematic variations in their lengths. The amplitude of the periods is of equal importance but for various reasons can be less satisfactorily determined. Necessarily the amplitude of period shorter than the 11-year period, derived from the relative numbers, must vary with the position in the 11-year cycle, since the scatter of the relative numbers as measured by the month-to-month variations is least at minima and greatest at maxima. Other solar data, however, have short period fluctuations with amplitudes equal to that of the 11-year variation in the data.

The writer's earlier paper (**2**) referred only briefly to the 28-month *solar* periodicities, and the present paper is intended to present additional evidence regarding the 11-year and 28-month variations in the mean heliographic latitude of the spotted area and other features of solar activity.

MATERIAL AVAILABLE FOR ANALYSIS

Solar data.—The solar data available for analysis are mainly the Wolfer series of epochs of maxima and minima beginning with 1610, and the relative numbers which comprise approximate yearly values from 1700 to 1749 and monthly values from 1749.

The Wolfer relative number is derived by the formula, $r=k(10g+f)$, in which g is the number of groups and isolated spots observed and f the total number of spots which can be counted in these groups and singly, while k is a coefficient depending upon the observer and his telescope. The relative number is, therefore, an index of the relative frequency of spots and groups.

Other solar data includes measurements of areas and positions of sun-spot groups from a projected image or a photograph. The Carrington and Spoerer series of solar observations give areas and positions of spot groups from 1854 to 1890. The Greenwich measurements of daily photographs from 1874 give the heliographic latitude and longitude of each group of spots with its area in millionths of the sun's visible hemisphere. These daily measures are combined into means for each rotation period and for the calendar year.

The Lyons Observatory, beginning with 1889, using a projected image of the sun, has published measurements of areas and positions of sun-spot groups, with means by calendar months. These data agree satisfactorily with the Greenwich data.

Other observatories such as Rome, Catania, and Kodaikanal have published various solar data including the mean daily number of groups, separate spots and pores while the Observatory of the Ebro has since 1910 daily measured the areas of the calcium flocculi.

Statistics of faculæ and prominences are other solar data available but since these two features are confined to a small area at or near the solar limb, their occurrence necessarily gives only incomplete information regarding the solar activity.

It is to be borne in mind that only one-half of the sun's surface is visible at any one time. Since the spots are practically the only available indication of solar activity, the daily measurements furnish only one-half of the total external evidence of activity. As a result there should be expected more or less irregularity in the various solar periodicities, since one-half the data is lacking. This irregularity will be especially noticeable when spots are few, and it is evident that the shorter the period the less regular the variation is likely to be.

The 11-year period is probably not affected greatly by this deficiency in the data as far as the total spottedness is concerned. The areas by hemispheres are likely to show considerable irregularity. The 28-month period is much more difficult to trace in the relative numbers and the still shorter periods are even more obscure. Fortunately the ratios of the spottedness of the two hemispheres seem to show the periods with greater regularity than do the total areas or the relative numbers, at least for the shorter periods.

The hypothesis of the continuity of solar periodicities is the one which seems to fit the facts most satisfactorily. Although they are difficult to trace at times owing to the unavoidable lack of complete data, their continuity and the strongly systematic character of their variations enable one to determine satisfactorily epochs even where the evidence is uncertain. In such cases it must be considered that if complete data were available the uncertainty would disappear.

Magnetic data.—Still another index of solar variability is the magnetic activity of the earth which synchronizes closely with solar activity. The 11-year variation in sun-spot numbers is strikingly reproduced in certain magnetic data, particularly the solar diurnal range of declination, and the shorter variations in these phenomena are also more or less highly correlated.

These various kinds of solar and magnetic data are strictly independent of each other and disclose widely different aspects of the sun's activity. They are of varying degrees of accuracy, depending largely upon the judgment of the observer and the local conditions of seeing and equipment. Only the Greenwich solar measures and certain magnetic data can be said to be independent of human judgment. Even magnetic data have a terrestrial element which renders it difficult to identify common features in them which may be correlated with solar variations.

Data analyzed by graphical methods.—The method of analyzing data employed by me is largely an empirical one. The probable existence of a periodicity is shown by a preliminary examination of smoothed graphs and a tabulation of the frequencies of the intervals between

the more pronounced crests and hollows. Tentative epochs of maxima and minima are then selected directly from the graphs and their reality confirmed by the aid of various criteria which are distinctive of the methods employed.

A principle which has been adopted as a working hypothesis in the selection of epochs is that the intervals between adjacent epochs should be as uniform as possible and that variations in the successive intervals should be in accord with the principle of continuity. There should be no abrupt change from very short to very long intervals. This is a reasonable working hypothesis, and it was found in the course of the investigation that this hypothesis comes closer to fitting the facts than any other. Of course, it must be realized that individual intervals are subject to rather large errors of estimation due to the rough method of selection and the effect of other periods, but a smoothing of successive intervals by moving averages of three terms will show that the changes occur gradually.

This is equally true of physical variations which are *not* periodic.

In endeavoring to determine by graphical methods any periodicities which may exist in a body of data, as for example the Wolfer relative numbers, it is highly advantageous to prepare graphs not only of the original data but also of various combinations of the data that may be derived by simple arithmetical methods. By a judicious treatment of the data it may be possible to discover relations that are only obscurely apparent in the original data. Perhaps the simplest illustration of treatment of data herein recommended in order to disclose more clearly periodic phenomena is Wolfer's method of adjusting monthly values of the relative numbers by a simple smoothing formula in order to eliminate irregular variations of shorter duration than a year, thereby facilitating the determination of the date of an epoch to the tenth of a year. Further illustrations of such treatment of data will be given below.

Again we may obtain from a single series of data two or more quasi-independent curves showing similar systematic variations with corresponding maxima and minima, which, however, may be unequally developed and even nonsynchronizing as to phase. If there is an underlying periodicity, it should be apparent in each of the curves, although not necessarily in equal distinctness.

THE 11-YEAR PERIOD IN SOLAR PHENOMENA

The various solar phenomena more or less directly associated with the sun spots, as the faculæ, flocculi, and prominences, have 11-year maxima and minima closely synchronizing with like phases of the spots.

A dominant feature of solar activity apart from the variations in the area and the frequency of spots is the 11-year variation in their latitude, manifest in the shift from high to low latitude and from one hemisphere to the other.

The mean latitude of spots has an 11-year period of variation. According to Spoerer's law the mean yearly values of the latitude are highest, around 20° to 25°, about a year after the epoch of minimum spottedness and lowest, around 6° to 8°, about a year before the following epoch of minimum spottedness.

Relation between the two solar hemispheres.—When we consider the two hemispheres separately we find that while they show roughly the 11-year period in their spottedness their epochs of maxima and minima are not coincident but differ at times a year or more. I have derived these epochs from the Carrington and Greenwich series of yearly values of frequency and spotted areas for each hemisphere. Minor irregularities were eliminated by consecutive 3-year smoothing which made it possible to determine the epochs to one or two tenths of a year by simple inspection of the smoothed values. From 1854 to 1870 the epochs for the southern hemisphere preceded those for the northern, the maximum difference being 0.5 year. At the maximum of 1870 the two epochs coincided. Since 1870 the northern epochs have preceded the southern epochs with a maximum difference of 1.2 years and an average difference of 0.5 year. For the whole series, 1854–1923, the average lag of the southern hemisphere has been 0.3 year.

The extent to which the two hemispheres vary together is indicated very clearly by correlating their mean variations for time units varying from one month to three years. The following coefficients have been computed by the method based on variate differences which has been explained in my former papers. Three-year means, 1874–1924, +0.91. Yearly means 1874–1924, +0.65. Five-rotation means, or 136 days, +0.37. 27-day means, 1874–1889, ±0.00. Monthly means, 1908–1924 (Lyons Obsy.) −0.10.

Evidently for short periods, as a month, the correlation is around zero. As increasingly longer units of time are employed, the positive correlation increases until, with minor year-to-year irregulatities eliminated, the 11-year variation common to each hemisphere yields a correlation above +0.90. It is clear therefore that the spotted areas of the two hemispheres vary in absolute independence of each other during short periods of time. Obviously, the investigation of total spottedness for evidence of short periodicities is likely to be disappointing.

The mean heliographic latitude.—There is, however, a method of combining the data from the two hemispheres which I have found to yield very definite results. The Greenwich reductions give for each day the latitude and area of each group of spots. The group latitudes are given weights proportional to their areas and thus there is derived a weighted mean latitude for each day which is designated as the mean heliographic latitude of the entire spotted area. These daily values are combined into means for each rotation period and for the calendar year, and have been used in what follows. The calendar year means are given in Table 1, column 4. For convenience these mean values will hereafter be designated as H. L. values. North H. L. is regarded as plus and south H. L. as minus.

Around epochs of minima there are frequently months with a total absence of spots. Obviously the H. L. has no value in those months and they are disregarded in combining monthly values into half-yearly or yearly means. This gives rise to considerable irregularity in periodicities of three years and under, at spot minima. Flocculi are observed continuously even when spots are absent and very satisfactorily supplement the spot data at such times.

The calendar year means of the H. L. are shown in Figure 1, curve 1, and the values, smoothed by the formula $(a+2b+3c+2d+e)\div 9'$, are shown as a dotted line. This curve shows a well-defined 11-year period in which the H. L. is farthest north three to four years before the maximum of spots and farthest south about a year or two before the minimum. The mean range of this 11-year variation in the mean H. L. is about 6°.

Ratio of northern to total spotted area.—Another method of showing this period is by computing yearly values of the percentage of spots in the northern hemisphere rela-

tive to the spottedness of the entire area. These percentages are given in Table 1, column 3. Curve 2 in Figure 1 shows the actual values and smoothed values are shown by the dotted line.

It is evident from the similarity between curves 1 and 2 that the percentage of spotted area in the northern hemisphere, derived by a simple operation, yields results similar to those disclosed by the mean heliographic latitude which is obtained only by a laborious computation.

Results of Carrington's and Spoerer's observations.—The observations by Carrington and Spoerer of the number and mean latitude of spot groups form a continuous series from 1854 to 1894, and enable us to trace back this variation in the H. L. 20 years before the beginning of the Greenwich record. The overlapping period of 20 years, common to the two series, serves to confirm, at least for the shorter variations, the general accuracy of the Spoerer record, which of course is not complete owing to lapses due to cloudiness, and is not strictly homogeneous with the Greenwich record, being a record of frequency, while that of Greenwich refers to the spotted area.

Beginning with 1854 there are available mean frequencies and latitudes of groups for each hemisphere with the mean H. L. The yearly values of the mean H. L. and the percentages of spot frequency in the northern hemisphere are given in Table 1, columns 1 and 2, and shown as curves 1 and 2 in Figure 2.

The 11-year variation shown by the smooth curve is clearly evident, the mean H. L. being farthest south at or shortly before a minimum of spotted area. The epochs of extreme southern displacement are 1857.5, 1868.0, 1877.0, 1889.0, 1899.5, 1911.5, 1920.5. It is noteworthy that the two belts of spots in the two hemispheres are nearest the equator when the southern hemisphere is relatively more spotted than the northern hemisphere, at or near the above epochs of extreme southern H. L.

THE 28-MONTH PERIOD IN SOLAR PHENOMENA

The 28-month period in the H. L.—It is evident from curves 1 and 2, Figures 1 and 2, that a short fluctuation of considerable amplitude is superposed upon the 11-year variation. That it is not a purely fortuitous fluctuation is seen from an analysis of the frequencies of the intervals between successive crests and hollows of curve 1, employing the annual means from 1874 to 1925. The frequencies, in percentages, of the 2-year, 3-year, etc., intervals are given below, with the probable frequencies for random data (3). The total number of variates is 31.

Interval in years	2	3	4	5	6
H. L. data, per cent	20	47	23	7	3
Random data, per cent	40	33	17	7	2

In purely random data the two-year interval is the most frequent, while in the H. L. data the three-year interval is the most frequent, indicating markedly systematic characteristics and consequently the existence of a tendency to recurrence. The length of this short variation ranges between two and four years.

In order to determine more precisely the main features of this short variation the mean H. L. was computed for each half year from the Greenwich mean values by solar rotation periods. The approximate mean values for the months January to June, inclusive, and July to December, inclusive, were thereby determined. These means were given in my paper (2) on the 28-month period, page 436, previously referred to, and are reproduced in Table I, columns 5, 6, and shown as curve 3 in Figure 1 with a free hand curve drawn to indicate the general trend of the short 28-month fluctuation.

A similar curve 4 can be derived from the values of the areas in the northern hemisphere, expressed as a percentage of the entire spotted area. Curves 5 and 6 show the Greenwich data as a frequency exhibit with the number of months in each half-yearly period having a north H. L., or an excess of spots in the northern hemisphere, expressed as a percentage of the total number of rotation periods, six or seven, which were combined into the half-yearly means. These two curves resemble each other rather closely, as should be expected, and are also highly correlated with curves 3 and 4.

The short fluctuation is shown to better advantage on eliminating periods of longer duration, as the 11-year variation. This is accomplished by taking residuals from the H. L. data smoothed by the formula $(a+2b+2c+2d+2e+2f+g)\div 12$. These residuals, 96 in number, further smoothed by the formula $(a+2b+c)\div 4$, are given in Table, 1 columns 7, 8, and shown graphically as curve 7, Figure 1.

The markedly systematic character of this curve is evident from the following tabular presentation of a comparison of the relative frequencies of the intervals between consecutive maxima or minima with those of random numbers (3) smoothed by the formula $(a+2b+c)\div 4$. The total number of variates is 40. The unit data are half-yearly means.

Interval	2	3	4	5	6	7	8
H. L. data, per cent	0	11	31	34	11	8	3
Random data, per cent	3	30	24	18	13	8	4

The most frequent interval for random numbers is the 3 interval, while for the solar data it is about 4.75 or 2.38 years since the unit data are half-yearly means. This frequency tabulation furnishes a better value of the probable length of the sequence than the previous tabulation which was based on annual means.

Correlating these H. L. residuals, both smoothed and unsmoothed and the same data with successive half-yearly shifts, there are obtained the following coefficients. The number of variates is 100.

Lag in years	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Smoothed r	+1.00	+0.36	−0.76	−0.52	+0.46	+0.48	−0.18	−0.14	−0.11	+0.49	+0.19
Unsmoothed r	+1.00	−0.50	−0.34	−0.20	+0.23	+0.37	−0.31	−0.39	+0.18	+0.42	−0.19

The maximum positive coefficients are at about 2.25 and 4.5, indicating a length of period of about 2.25 years. The coefficients from the unsmoothed data are less regular than from the smoothed data and the maximum at 2.5 is smaller but the phases of the two sequences are substantially coincident. Obviously the value for the recurrent feature, 2.25 years, is not due to the smoothing process.

In order that the reader may realize that the maximum positive coefficients, around +0.48, for the first and second recurrences at 2.25 and 4.5 years are really relatively large and undoubtedly indicate the existence of a recurrent feature in the data, a comparison with a similar self-correlation of the yearly relative numbers from 1750 to 1925 is given. For this period of time, 175 years or about 16 recurrences of the 11-year period,

the series of coefficients for varying lags up to 25 years shows the first maximum positive value +0.59 at 11 years and the second +0.48 at 23 years. Thus for the 11-year period whose existence is well established, we have a coefficient for the first recurrence not much greater than that for the short period. Dividing the sunspot data into two parts and correlating each part separately we find for the first half, 1750–1829, +0.48 as the maximum coefficient at 10 years, and +0.70 as the maximum coefficient at 11 years from 1830 to 1925. The large variability in the length of the period in the first half of the series accounts for the low coefficient.

These values of the probable length of the period obtained by the method of self-correlation and the frequency tabulation of the intervals between crests and hollows are, of course, only rough approximations. The average length, like that of the 11-year period, is derived from the epochs of maxima and minima, selected after a careful study of curves 3–7, Figure 1. This selection of the epochs is facilitated to a certain extent by a knowledge of the approximate value of the period length

The Carrington and Spoerer series of observations from 1854 to 1894 are also combined in periods of five solar rotations, making about 2.5 values per year. These observations comprise the number of groups and their mean latitude for each hemisphere. I have derived the mean H. L. for each of these periods by computing for each hemisphere the product of the number of groups and the mean latitude. The difference between the two products divided by the total number of groups gives the H. L. These H. L. values, smoothed by the formula $(a+b)\div 2$, are shown as curve 3, Figure 2, with a free-hand curve drawn to indicate the trend of the 28-month variation. The 20-year overlap in the two series shows substantial agreement between records based on frequencies and on areas.

The definitive epochs of maxima and minima derived from the H. L. curves are given in Table 2, columns 1, 2.

The following table shows the frequencies of the H. L. intervals between maxima and minima, Table 2, column 3, expressed in percentages of the total number of variates, 53. There are given for comparison a frequency tabulation of the Wolfer 11-year intervals, in percentages of the total number, 55.

H. L. 28-month intervals		Wolfer 11-year intervals	
Years	%	Years	%
1.50	2	7.4	2
1.75	4	8.3	9
2.00	15	9.2	9
2.25	15	10.1	18
2.50	30	11.0	24
2.75	9	11.9	15
3.00	11	12.8	7
3.25	2	13.7	11
3.50	8	14.6	4
3.75	4	15.5	0
		16.4	0
		17.3	2

It is clear that there is little difference in the two series as far as variability in the intervals is concerned. The Wolfer intervals range between 7.3 and 17.1 years, the most frequent interval being 11 with 24 per cent. The H. L. intervals range between 1.50 and 3.75 years, the most frequent interval being 2.50 with 30 per cent. The five intervals, 2 to 3, comprise 80 per cent of the whole number. The average interval is 2.56 years. The distribution, like that of the Wolfer intervals is unsymmetrical with a positive skewness, the mode being less than the mean. The mean deviation of the intervals from 2.5 years is ±0.38 year.

The 28-month period in the relative numbers.—The 28-month period, while clearly evident in the H. L. data, is discernible in the relative numbers only after the 11-year period has been eliminated. The simplest way to effect this elimination is to compute from the monthly numbers half-yearly means ending June 30 and December 31 for each year, and combine these into running means of five. Each of these latter means comprises therefore a period of 30 months, or about the length of the short period. Residuals of this smoothed series from the original data give the short period freed from the 11-year variation, or any other long-period variation. These residuals are given in columns 9, 10, Table 1.

When this is done, there appear secondary spot maxima and minima one to two years after the epochs of maximum north and south H. L., respectively. These spots of maxima and minima are given in Table 2, columns 5, 6. This lag is analogous to that of three to four years in the case of the 11-year variation. The lag, therefore, varies directly with the length of the period. Curve 8, Figure 1 is a graph of the residuals of the relative numbers.

For convenience the Wolfer relative numbers will be hereafter designated as S. S.

The S. S. curves, even after the elimination of the long period variations, do not show the 28-month period as regularly as do the H. L. curves. At times the maximum or minimum of spots corresponding to an H. L. epoch is not well defined. Since, as stated above, the correlation between the spotted areas of the two hemispheres is around zero, it follows that the correlation between the ratio of their spottedness, or the H. L., and the total spottedness must be relatively low. The general tendency, however, is toward a positive correlation, allowing for a lag in the S. S. data, varying with the length of the period.

The 28-month period in the interdiurnal variability of the relative numbers.—Another series of solar data is the interdiurnal variability of the Wolfer sunspot numbers or the mean variability of the daily relative numbers. The writer has computed monthly and half-yearly means of this element since 1873. This measure of the scatter of these numbers is a quasi independent series of data, coequal in importance with the monthly numbers themselves. When the 11-year variation is eliminated from these values by the same method employed for the relative numbers, there are obtained residuals which show a correlation of +0.81 with the S. S. residuals. These residuals are in columns 11, 12, Table 1, and are graphically shown as curve 9, Figure 1. The curve shows the 28-month variation even better than does the curve of S. S. residuals. The phases of the two curves are nearly synchronous, there being a lag of about 1½ months, on the average, in the phases of the S. S. curve. Correlating the S. S. residuals with the variability residuals six months earlier, coincident with, and six months later, there are obtained coefficients, −0.10 +0.81, and −0.45, respectively, which conform the existence of the small lag in the S. S. residuals.

The 11-year variation in the length of the period.—A well-defined systematic variation in the length of the 28-month period is clearly evident from an analysis of the H. L. and the S. S. epochs in Table 2. The columns following the epochs give the intervals between consecurive maxima or minima and in columns 4 and 8 are given the intervals smoothed by the formula $(a+b+c)\div 3$. These smoothed intervals are shown in Figure 3, curves

1 and 2, plotted opposite the dates midway between the epochs of maxima and minima. The 11-year variation is evident in these series of intervals but is more regular in the curve derived from the relative numbers. Table 3 gives the 11-year epochs of maximum and minimum length of the period, from which it appears that there is a persistent opposition of phase between the two curves, maxima and minima, of the H. L. curve corresponding to minima and maxima, respectively, of the S. S. curve.

The epochs of long and short 28-month intervals, derived from the relative numbers, occur near the Wolfer epochs of sunspot minima and maxima, respectively, and precede them with an average interval of 0.6 year. In other words, near epochs of sun-spot maxima the solar activity increases and one result of this increased activity is a decrease in the length of the short period of 28 months. An analogous relation is seen in the relatively short interval between a very intense 11-year maximum phase and the preceding minimum.

The 36-year variation.—There is also evident from a careful study of the graph (Fig. 3, curve 1) a 36-year variation in the length of the short period, long intervals occurring about 1860 and 1893, and short intervals about 1875 and 1912.

The lag in the epochs of spottedness.—It was shown above that the mean H. L. is farthest north 3 to 4 years before the Wolfer epochs of maximum spottedness. A similar lag in spottedness is evident in the 28-month variation. Table 2, columns 13, 14, gives the lag or the interval from the epochs of H. L. to those of S. S. The average lag from 1855 to 1925 is 1.35 years. However there are marked systematic variations in the lag, an 11-year variation being very evident. These lags smoothed by taking means of three are shown as curve 3, Figure 3, from which are derived the 11-year epochs which are given in Table 3. These epochs of long and short lags average about 2 years after the Wolfer epochs of minima and maxima, respectively. The 36-year variation is also evident, with maxima around 1857 and 1890 and minima around 1874 and 1905.

Continuity and independence of the 28-month period.—A careful examination of curves 8 and 9, Figure 1, should convince one of the unbroken continuity of this secondary variation of solar activity. Freed from the 11-year variation, the epochs furthermore show no relation whatever to the Wolfer epochs. The two periods are incommensurable with and independent of each other.

This is further evident on referring to the graph of actual and smoothed relative numbers in the MONTHLY WEATHER REVIEW, April, 1902. At certain 11-year maxima, as 1770, 1803, 1830, 1870, 1883, there are two short-period crests about two years apart with not greatly differing ordinates; likewise at certain 11-year minima, as 1755, 1798, 1823, 1877, 1888, 1912, there are two nearly equal scondary minima, about two years apart, separated by a secondary maximum.

A new determination of the 11-year epochs of maxima and minima.—It is clear from this graph that these secondary fluctuations have a considerable amplitude around sunspot maxima and that by employing a smoothing formula of only 12 terms, as Wolfer did, the 11-year maximum epoch of spottedness is made to coincide frequently with a secondary maximum of spottedness. The logical inference is that the short period must be properly eliminated in order to obtain the most probable date of the 11-year epochs. Wolfer's epochs were read off from a composite curve and do not truly represent epochs of the elemental 11-year variation.

In order to eliminate this short period I have employed the usual method in such cases, that of smoothing the data by combining into one average successive portions of the data having a length equal to the period to be eliminated. I have employed as the unit data 12-month means, six months apart, and centered on January 1, and July 1. From these means consecutive means of five were computed and plotted in a curve having two means per year, from which the 11-year epochs can be readily estimated to one or two tenths of a year.

These epochs are given in Table 4. The variation from Wolfer's epochs is in most cases small, only 5 out of 32 epochs showing a deviation of six months or more. The largest difference is at the maximum of 1804 where it is 1.1 years earlier than Wolfer's and the long period between the maxima of 1788 and 1804 is 15.8 years instead of 17.1 years. The probable error of the intervals maximum to maximum and minimum to minimum is ± 1.2 years for the Wolfer epochs and ± 1.1 years for the new epochs.

These epochs are not to be regarded as in any sense superseding the Wolfer epochs. They are merely epochs derived by the application of a different smoothing formula. The 11-year epochs of magnetic declination range referred to below were derived in the same manner, as it was desirable to completely free each series from the short-period fluctuations in order that the two series of 11-year epochs might be strictly comparable.

ANALYSIS OF MAGNETIC DATA

We are not limited, however, to solar data for evidences of the 11-year and 28-month variations. It has long been known that the 11-year variation in spottedness is closely paralleled by the variation of the diurnal inequality range of magnetic declination.

The diurnal inequality of declination for any particular month is the sequence of the residuals of the means of the hourly values of declinations from the monthly mean declination. On the average of quiet days in the summer months there is a smooth progression of the residuals with extreme values around 7 to 8 a. m. and 1 to 2 p. m.

The 11-year epochs of maxima and minima.—In the year 1852 Wolf, Sabine, and Gautier published nearly simultaneously their discovery of a direct and parallel relation between the solar spottedness and the mean daily range of magnetic declination. Wolf derived a formula connecting yearly values of magnetic range, R, and sunspot frequency, S, by a relation $R = a + bS$, in which a and b are constants to be determined from the observed values of R and S. He compiled in his Astronomische Mitteilungen all available magnetic data and showed that this close relation has existed since 1780, and derived epochs of maximum and minimum range. Making due allowance for accidental deviations in individual cases, the coincidence of the 11-year epochs of the two phenomena is so close that the magnetic data can be regarded as coequal with solar spottedness as an index of solar activity.

I have made a new determination of the 11-year epochs of maximum and minimum declination range, using for this purpose the table compiled by Fritz **6** which gives all available values of yearly ranges from 1834 to 1877 for numerous stations. Most of the data previous to 1873 are based on eye observations at hours selected to yield approximately the extremes, usually 8 a. m. and 2 p. m. If fixed hours are consistently adhered to, the epochs derived therefrom are quite as reliable as those from recording instruments. Numerous sources of error, however, tend to render the series at certain stations non-

homogeneous in many cases—change in instruments and hours of observations, spider webs or fungous growths causing a restricted range, etc.—but by averaging the results for a number of stations the desired accuracy is attained.

For the derivation of epochs subsequent to 1873 practically all published records from stations having magnetographs have been examined. Monthly mean values of the range between the minimum around sunrise and the early afternoon maximum have been tabulated for these stations, and the data are therefore, comparable with the Fritz yearly ranges. It is believed that higher correlations between the sunspot relative numbers and magnetic data will result from the employment of magnetic ranges thus derived than from the 24-hour ranges which are commonly regarded by magneticians as the diurnal inequality range.

The method of deriving the magnetic epochs is in all respects similar to the one employed for the derivation of the epochs of sunspot maxima and minima. Yearly ranges around the epochs for a number of stations were averaged and either by simple inspection of the data in the case of a single well-marked crest or hollow or guided by a preliminary smoothing, epochs to the nearest quarter of a year were selected.

Table 4 gives the epochs of maximum and minimum diurnal inequality range. The average deviation from the sunspot epochs is + 0.08 year, the solar epochs preceding the magnetic epochs.

The 28-month epochs.—As stated above there are secondary fluctuations superposed upon the primary 11-year variation in the diurnal inequality range. Employing on the monthly ranges the same smoothing process by which Wolfer derives his smoothed relative numbers, the annual variation and periods of shorter length are eliminated. The early numbers of Wolf's Astronomische Mitteilungen contain such smoothed monthly values of declination range for 6 to 10 stations in Europe for each year from 1839 to 1890. I have plotted these values and derived independently for each curve the 28-month epochs of maxima and minima. For various reasons above mentioned there are minor divergencies between the curves, but there is little difficulty in determining by a comparison of all the curves mean epochs within an accuracy of one or two tenths of a year, due consideration being given to the effect of the large amplitude of the 11-year variation in displacing the position of the epochs. Most of Wolfer's data were based on eye observations but the ranges were derived from hours around 8 a. m. and 1 p. m. and the epochs therefore are approximately those of the diurnal inequality ranges and may be regarded as substantially accurate.

The only available series of hourly observations from which the diurnal inequality range, 8 a. m. to 1 p. m., can be derived previous to 1870 is that at St. Petersburg from 1841 to 1862. Greenwich 24-hour ranges are available from 1841 but comparison with other stations shows numerous irregularities in the series which renders them unsuitable for this purpose.

I have tabulated the monthly diurnal inequality ranges for the following stations: Greenwich, 1868–1925; Pavlovsk, 1873–1908; Tiflis, 1870–1908; Paris, 1883–1925; Potsdam, 1890–1924; Pola, 1886–1915; San Fernando, 1880–1924; Bombay, 1871–1905; Mauritius, 1883–1909 (broken); Cheltenham, 1901–1924; Sitka, 1902–1924.

For each of these series I have computed two 12-month means per year, centered January 1 and July 1. These values were smoothed by moving averages of five terms to eliminate the short variation. Residuals of these smoothed values from the actual values give the 28-month variation freed from the 11-year variation. From graphs of these residuals epochs of maxima and minima were read off to the nearest quarter of a year. Figure 4 shows a number of these graphs. Table 5 contains these epochs with the corresponding epochs of Wolfer's relative sunspot numbers. The differences between the two series range between +1.8 year and −1.0 year. The mode is 0.0, the probable error is ± 0.54 year and 57 per cent of the intervals are 0.25 year or less. The mean is +0.068 year, the solar epochs preceding the magnetic epochs. This lag in the magnetic epochs is apparently real since it is the average of 75 cases, which seems a sufficiently large number to insure elimination of accidental errors in the individual epochs.

These magnetic epochs have been derived almost exclusively from records in the northern hemisphere. Since the diurnal inequality range at the winter solstice is very small at stations in high latitudes, the variations are obviously mainly determined by the ranges of the warmer months. Data from the southern hemisphere should be available to obtain variations in December and adjacent months to supplement those in the other hemisphere. This is the chief reason for the occasional large deviations from the sunspot epochs. Then, too, the latter epochs are subject to much uncertainty for various reasons. The two series of epochs, however, satisfactorily agree in the long run and with more data available and in a more homogeneous form they should agree closely at all times. Magnetic data give independent evidence of great value for the determination of the 28-month variation and should be available to students as early as possible.

THE 28-MONTH PERIOD IN THE UNITED STATES TEMPERATURES

The epochs of maximum and minimum temperature for the United States are given in Table 2. They are substantially as published in my earlier paper on the 28-month period. A curve of the smoothed intervals between consecutive maxima or minima is shown as curve 4 in Figure 3. There is evident a tendency to an 11-year variation in the length of the period; long and short periods occur about four years after the Wolfer epochs of minima and maxima, respectively. The 11-year epochs of long and short intervals are given in Table 3.

The association of minima of temperature with the epochs of maximum north H. L. is regarded as the true relationship. This is based on statistical and other considerations and will be discussed below. Table 2 columns 15, 16 contains the lag of the temperature epochs relative to the H. L. epochs. These lags smoothed are shown as curve 5, Figure 3. The average lag is 1 year with, however, a systematic variation in length having a period of 11 years. Long and short lags occur about five years after the Wolfer epochs of minima and maxima, respectively.

When the epochs of temperature were published the H. L. epochs from 1855 to 1875 were not known. The reality of these latter epochs is confirmed by the same consistent relationship between them and the H. L. epochs that has characterized the two series of epochs from 1875 to 1925 and which was graphically shown in Figure 1 of my earlier paper. (2.)

Relation of short-period variations in temperature to solar conditions.—The question arises what are the par-

ticular solar phenemona with which the terrestrial temperature variations are most closely associated? Do they depend on variations in spottedness or upon the variations in the position of the spotted areas? The 11-year temperature variation seems to be closely related to the variations in spottedness since opposite phases very nearly synchronize the phase differences being of the order of a few months only, when long records are analyzed.

On the other hand the 28-month variation in temperature seems to depend upon the position of spots rather than their magnitude or frequency. This is apparently shown by an analysis of the lag of the temperature epochs relative to various solar phases. The lag of the temperature epochs from the H. L. epochs, regarding epochs of temperature minima as associated with epochs of maximum north H. L., is shown in Table 2, columns 15, 16. The average of these is 1 year, the mean deviation, 0.35 year, and the mean variability 0.27 year. Incidentally, attention should be directed to the very low Goutereau ratio (cf. MONTHLY WEATHER REVIEW September, 1924, pp. 432, 441), which is 0.77, while for unrelated numbers it is 1.41. This small ratio indicates a highly systematic variation in the values of the lag, as is evident from the graph (5) in Figure 3.

When the lags of the temperature epochs from the sunspot epochs are analyzed, we have for the lag based on like phases, average 0.9 year, mean deviation 0.61 year, mean variability 0.32 year. In this case the Goutereau ratio is only 0.52. For the lag based on unlike phases, or the lag from maximum spottedness to minimum temperature we have, average 2.14 years, mean deviation 0.59 year, and mean variability 0.33 year. The fact that the scatter of the values of the lag is so much smaller when the H. L. epochs are used seems to indicate that in some way the temperature depends more on the position of the spots than upon their magnitude. At any rate, the forecasting of temperature variations from solar variations will show a greater probability of verification when based on the H. L. rather than on the S. S. epochs.

That the causal relation is between the maximum north H. L. and the immediately following tempeature minimum is shown by the greater mean deviation of the lags with other combinations. The lag from the maximum north H. L. epochs to the immediately following temperature minima has, as above stated a mean deviation of 0.35 year. From the same epochs to the next following temperature maxima the lag has a mean deviation of 0.40, while the lag to the second following temperature minimum has a mean deviation of 0.46 year. It seems reasonable that the causal relation is with the combination having the least mean deviation of the lag. The number of variates, 100, is sufficiently large to justify confidence in these values.

It is noteworthy that the mean deviation of the lag from the H. L. epochs is much less for the temperature epochs than for the S. S. epochs, which is probably due to the relatively low correlation between the spottedness of the two hemispheres referred to above.

Having presented the main results of the investigation a discussion of their significance will now be given and certain criticisms answered.

The point at issue is, of course, the reality of the 28-month epochs in Table 2. The author's paper (2) on the 28-month period has been criticized by some meteorologists seemingly from failure properly to apprehend and appreciate the significance and evidential value of the results obtained from the simple statistical methods there employed. It is charged that the selection of epochs from graphs must be on the basis of a minimum amplitude or some such criterion otherwise the selection must be influenced by one's desires. By reference to the curves of H. L. and S. S. residuals it will be seen that the employment of a criterion based on amplitude alone would be utterly futile. Such a method would be effective only when the amplitude is fairly uniform as in the case of the 11-year variation in sunspots, where amplitude is the only criterion necessary. The 28-month variation unfortunately is much less obvious and other criteria must be resorted to in order that the selection of epochs shall not be merely arbitrary.

To many readers the methods employed by me may seem loose and inefficient. Walker (4), in a discussion of my previous paper on the 28-month period in weather conditions deplores a "departure from definite and reliable methods based on Fourier analysis and in their place we have" (quoting words from my paper) "an empirical method based largely upon careful examination of curves drawn free hand through plotted data or derived by means of smoothing formulæ." He points out that the selection of epochs of maxima and minima, either from smoothed or unsmoothed curves, must be made on the basis of some definite criterion, "otherwise our selection must be influenced by our desires." Finally, in discussing the changing lengths, which range from 1.8 to 3 years, of the 28-month period shown by me to characterize the pressure at Portland, Oreg., he says:

> Now if surges of 1.8 years and of 3 years are treated as variations of a 28-month period, what limit is there to such variation, and why should we speak of such surges as constituting "periods"? The word "period" has hitherto had a definite meaning in physical mathematics and it will tend to confusion if it has to bear also a second meaning.

While recognizing the abstract superiority of rigorous methods like the periodogram analysis, and indeed the self-correlation of a series of quantities after they have been shifted successively one, two, three, etc., time steps with reference to each other, nevertheless these analytically similar or equivalent rigorous methods are peculiarly adapted to periodicities which are of uniform length, and their application has in general failed to give definite indications of periodicity in meteorological and solar data. This is a result to be expected if real periods exist which vary considerably in length, especially when many repetitions of the irregular periods are comprised in the computations. Moreover, when the results secured by both the rigorous and the empirical methods are fairly interpreted the findings are not seriously in conflict.

In his paper Walker applied the self-correlation method to pressures at Darwin which he considers are representative of a "wide-spread belief * * * of a 3 to 3½ years' period in the region extending from Java to N. Australia." This method was employed by me in my paper on the 28-month period, using an abridged method of correlation based on variate differences, which, however, when the number of variates is large, yields results essentially the same as by the usual Pearsonian method. Walker correlates the Darwin pressure data for each season and the entire data and finds for the latter a broad band of positive correlation coefficients from 2½ to 4½ years with the maximum coefficient $+0.26 \pm 0.05$ at approximately 3⅛ years. The reality of the period is apparently indicated by his criterion based on the probable maximum coefficient. The separate seasons, however, show a variation in the time interval which gives the

maximum positive coefficient. For December to February it is 2⅞; for March to May and September to November, 3⅛; and for June to August, 3⅜ years. The average of the four seasons is 3⅛ years, which agrees with the interval given by the entire data.

There is therefore a variation of one-half year in the apparent length of the period when the data are correlated for the separate seasons. This shows that when rigorous methods are applied to periodicities which vary in length the results may differ considerably when the data are correlated in various ways, a further illustration that all such results are to be interpreted with much caution.

The reality of this recurrence seems highly probable from a consideration of the various criteria. Its length, however, can be only approximately derived by such rigorous methods as the periodogram analysis and self-correlation. This is well illustrated by the sunspot period, which is 11.13 years when derived by a least square computation from the epochs of maxima and minima. The periodogram analysis, however, gives the maximum amplitude at about 11.4 years, based on the curve from 1750 to 1900. Similarly the method of self-correlation indicates for the Darwin and Batavia pressures a probable recurrence of 3 or 3⅛ years in length, while the average length derived by my methods from 1882 to 1920, according to my table of Batavia epochs, is 32.7 months, or nearly 2¾ years. The length of a variable period can be best obtained by first determining the epochs of maxima and minima.

To show that the method of correlation used by me gives essentially the same results as to length of period as the ordinary method used by Walker, I have computed the coefficients for the mean half-yearly pressures at Darwin for the same interval of time, 1882–1923, with the following results. The number of variates is 84.

Time steps, years	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Correlation: r Clough.	+1.00	+0.30	−0.47	−0.43	−0.47	+0.31	+0.45	+0.31	−0.46	−0.38
Walker.	-------	-------	-------	−.10	−.12	+.12	+.24	+.16	+.04	0

The sequence of the coefficients indicates a recurrence of 3 years as the most probable interval, which agrees closely with Walker's results, even though the coefficients differ numerically.

With reference to a criterion for the selection of epochs the writer believes this demand is satisfied by the great care exercised before the final epochs are adopted. Unfortunately the period is not clearly defined in any one curve, and it is necessary to compare curves of various elements at the same station as well as curves from neighboring stations, that the epochs may be consistent. This process was carefully explained in my paper on the 28-month period and seems to be a reasonable anticipation of and answer to Mr. Walker's objections. The explanation is quoted as follows:

> The observations at Batavia were studied in considerable detail. Three means per year were employed, each four-month period receiving an appropriate correction for annual variation. Curves were drawn for four elements, viz, pressure, mean maximum temperature, mean minimum temperature, and rainfall. By comparison of these four curves it was possible to determine definitive epochs since 1866. The epochs of maximum and minimum pressure coincide closely with the epochs of minimum and maximum frequency, respectively, of rainfall. The epochs of mean maximum temperature follow similar phases of the pressure by about four months as an average. The epochs of mean minimum temperature follow by a few months the epochs of the mean maximum temperature. The fluctuations of pressure are entirely representative of such fluctuations for the whole Indian Ocean region, including India, Australia, and Mauritius.

Curves, therefore, for four elements at the station were compared and epochs so selected as to insure consistency in them as regards the mentioned interrelations and lags, which are obvious from simple inspection of the curves. Furthermore, curves for many other stations in the Indian Ocean region were examined before definitive epochs were finally adopted. There are many factors involved in meteorological variations and it is only by a careful comparison of numerous curves for different stations that the significant and real features emerge from the accidental features which may partially obscure them in the individual curves. An illustration is the anomalous depression in the maximum temperature curve at Batavia in the latter part of 1883 which is plausibly due to the Krakatoa eruption.

Again, the epochs of pressure and temperature for Europe were selected so that for each epoch of pressure in southwestern Europe there was a corresponding epoch of temperature with like phase in northern Europe. This relation is a necessary consequence of the association of high temperature with winds having a southerly component and vice versa.

Taking up finally the application of the word "period" to recurrences at intervals which vary in such ratios as 1.8 to 3, the writer realizes such a practice is objectionable; however, up to the present time no good substitute word has ever been proposed. Moreover, repeating a statement at an earlier place in this paper, astronomers have long been accustomed to designate as periods such irregular recurrences as that of maximum solar spottedness and the maximum brilliancy of variable stars. We may, therefore, regard this practice as an instance of a meaning inconsistent with the primary significance of a word but sanctioned by usage. The solar cycle of 11 years varies between 7 and 17 years, one-half of the intervals being less than 10 years or greater than 12 years, and the variations can not be satisfactorily explained on the basis of fortuitous causes alone. This being the case the author believes he is justified in claiming to find evidences of variability in the various solar and meteorological periods

The evidences for the reality of the results of the investigation are mainly of a statistical nature and obviously some familiarity with statistical processes and a certain amount of experience in their application to observational data is essential in order that the reader may fully appreciate the significance of the results.

The existence of the 11-year variation in various solar data is clearly obvious from the yearly means of spots, faculæ, flocculi, prominences, etc., and the mean latitude of all spots. Other data, as the mean heliographic latitude of the entire spotted area, require a preliminary smoothing to cearly disclose the 11-year variation. This, however, is an ordinary statistical procedure, similar to the method employed by Wolfer in the derivation of his smoothed monthly relative numbers.

The existence of the 28-month period, however, is not as obvious from simple inspection of a graph as is the 11-year period in a graph of the yearly relative numbers or the average latitude of spots. It is first necessary to establish by purely impersonal, rigid, statistical tests the fact that the order of sequence of the peaks and hollows of certain solar data and the most frequent interval between them differ markedly from that which would characterize them if their occurrence were perfectly fortuitous. Given the observational data, anyone employing the methods and criteria used by the writer will

necessarily obtain the same results as set forth on page 254. These results are absolutely devoid of any human element.

Tabulating the frequencies of the varying intervals between peaks and hollows of the smoothed residuals of the half-yearly means of the H. L. data we derive the most frequent interval, 2.4 years. With this datum we proceed to locate tentative epochs and draw a smooth curve, ignoring minor fluctuations. The next step is an intercomparison of two or more curves similarly derived to insure consistency between the epochs and confirm their reality, based on their a priori or statistically derived relationships.

variation averages 11 years, which is the period of variation of the spotted area, affords a substantial basis for regarding them as real.

Another significant relation is that the 28-month period in the relative numbers is shortest 0.6 year before the Wolfer epochs of maximum spottedness. A priori there would seem to be good reason for this since we should expect that at the time of maximum solar activity the various solar processes would be accelerated, and if there were short period fluctuations they would be completed in a relatively short period of time, just as un-

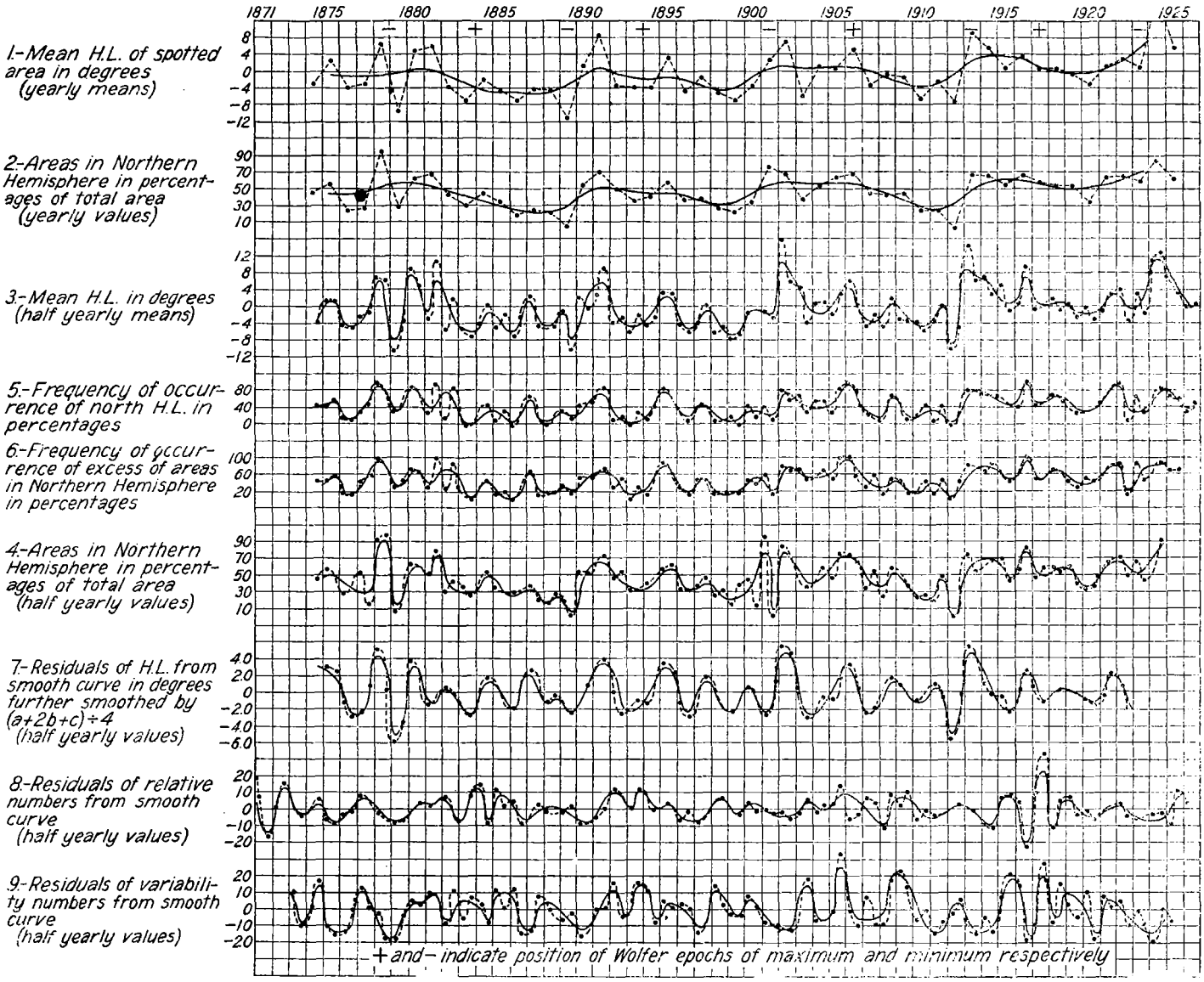

FIG. 1.—Yearly and half-yearly means of data derived from the Greenwich solar measurements and the Wolfer relative numbers, with smooth curves showing the 11-year and 28-month periods

The epochs of the H. L. and the S. S. having been selected, with of course an element of human judgment entering into them, further tests are applied in order to detect evidence of systematic tendency in them which would be wholly lacking if they were unreal. Thus the existence of an 11-year variation in the length of the period in both the H. L. and S. S. series of epochs (cf. curves (1) and (2), fig. 3) is obvious evidence that the epochs from which the period lengths are derived are as real as Wolfer's epochs. The fact that the long-period usually intense 11-year maxima follow very soon after the preceding epoch of minimum.

It was shown in my former paper (**1**) that when the sunspot period is around 8 to 10 years, high relative numbers at maxima occur a few years later.

It is evident, therefore, that the real measure of solar activity is the length of the solar periodicities, short intervals being associated with maximum activity and vice versa. The amount or frequency of spottedness appears to be a result rather than an indication of activity, since

the epochs of spottedness occur 0.6 year later than the epochs of the length of the period.

A still further significant result is the well-defined 11-year variation in the lag of the S. S. relative to the H. L. epochs. (Curve (3) fig. 3.) The two series of epochs were originally derived independently of each other. The existence of this regular variation in the differences between two series of epochs which were independently derived would seem to be utterly improbable unless the two series represented real phases of solar activity and unless the variations of the secondary fluctuations were actually conditioned by the fundamental period of solar activity, the 11-year period. Such regular systematic variations could not possibly result from data akin to random numbers which would necessarily characterize a selection of epochs based on a criterion of amplitude alone or in which a large element of human judgment or personal bias was involved.

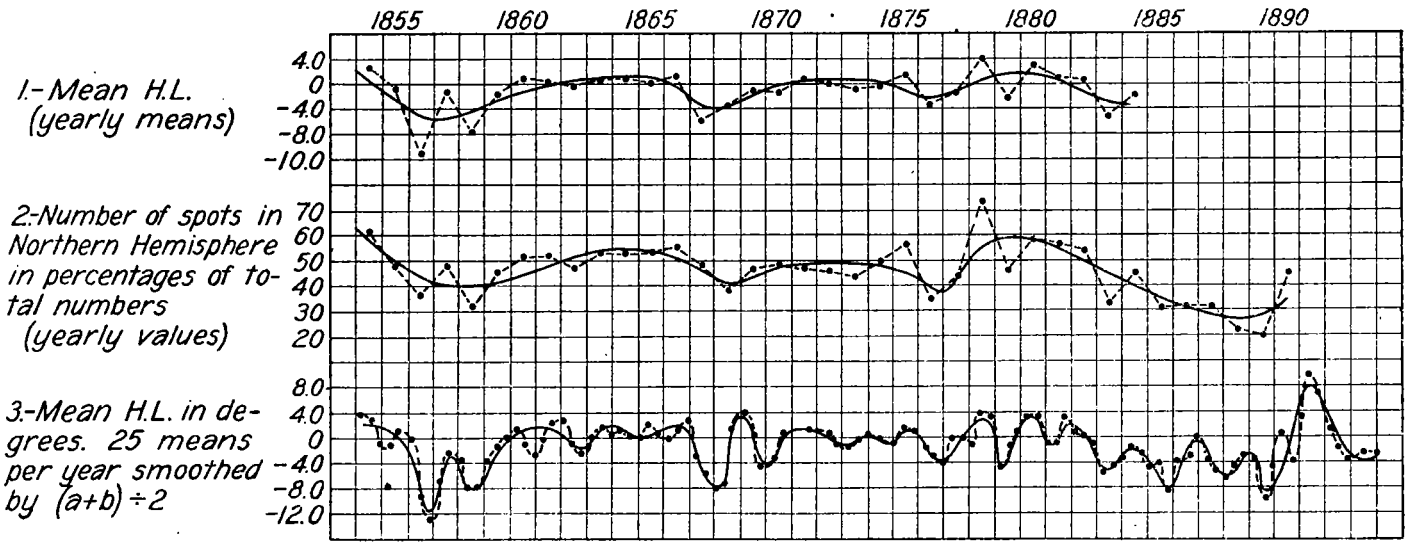

FIG. 2.—Yearly and half-yearly means of data derived from Carrington's and Spoerer's solar measurements with smooth curves showing the 11-year and 28-month periods

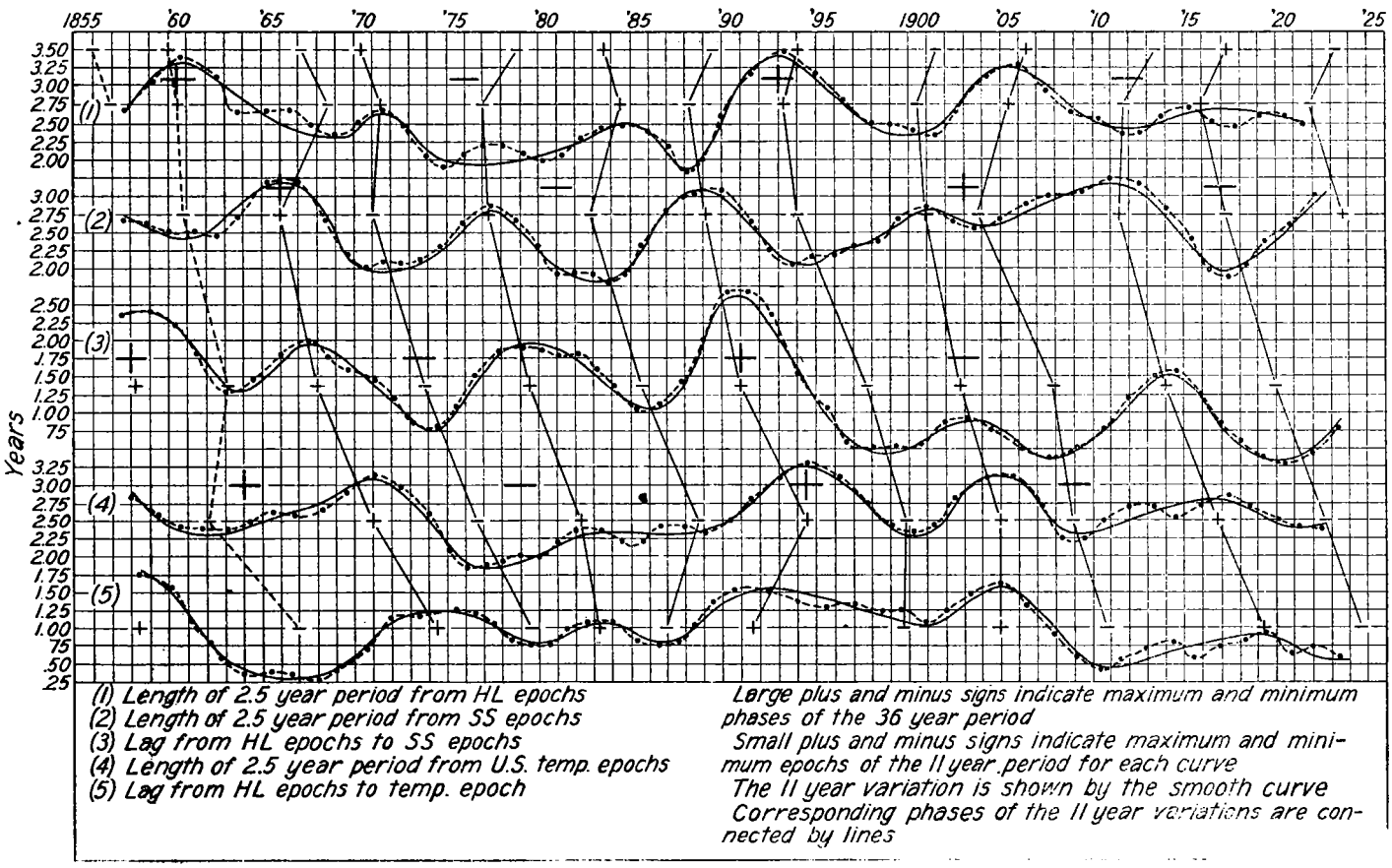

FIG. 3.—The 11-year and secular variations in the length of the 28-month period in themean heliographic latitude, relative numbers, and the temperature in the United States, and in the lag from the H. L. epochs

The opposition in phase of the variations of the short period derived from the H. L. and S. S. epochs (cf. curves (1) and (2) fig. 3) is another relation which is quite inexplicable from an apriori point of view, but its persistence over a period of 75 years again would seem to preclude any possibility of the unreality of the epochs.

Disregarding, the details of the methods used, the outstanding feature of the whole investigation is the consistency of the results obtained, *shown by the persistent appearance in the final results of the 11-year variation in Fig. 3.* Just as the meeting of two tunnel bores from opposite sides of a mountain is sufficient evidence of accuracy in the observations taken to insure correct alignment of each bore, so these consistent end results are prima facie evidence of the validity of the methods employed in their attainment.

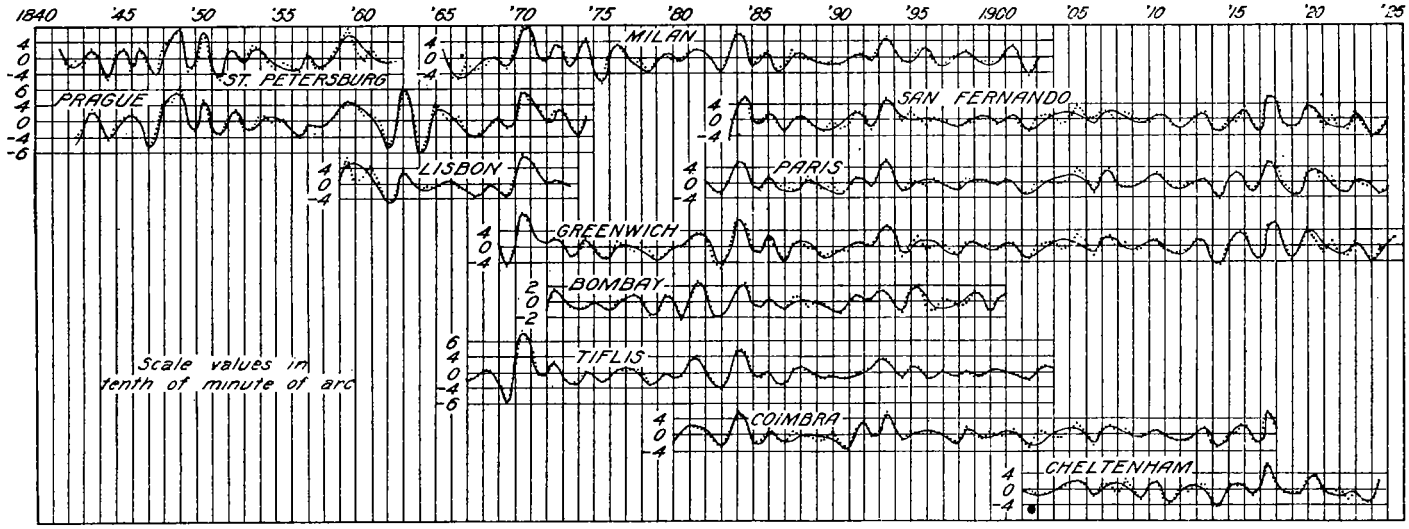

FIG. 4.—Diurnal inequality ranges of magnetic declination. Half-yearly means with 11-year period eliminated and smooth curve drawn to shown the 28-month variation

TABLE 1.—*Yearly and half yearly values of solar data*

[H. L. signifies mean heliographic latitude. S. S. signifies relative sunspot numbers. % N. signifies area of spots in northern hemisphere in percentages of total area. (1) and (2) are first and last halves of year]

	Carrington and Spoerer data		Greenwich data						Wolfer data			
	1	2	3	4	5	6	7	8	9	10	11	12
	% N.	H. L.	% N.	H. L.	H. L.		H. L. residuals (smoothed)		S. S. residuals		Variability residuals	
					(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
		°		°	°	°	°	°				
1854	61	+2.5	1874	46	−3.0		−3.7		0	+5	+2	+18
1855	49	−0.8	1875	56	+2.8	+1.6	+1.4	+3.0	+2.4	−6	−9	−10	−14
1856	37	−11.2	1876	26	−3.8	−4.1	−4.6	−1.1	−2.8	−4	−1	−13	0
1857	49	−1.4	1877	28	−2.7	−2.3	−1.6	−2.2	+0.9	+7	+1	+14	+2
1858	31	−7.8	1878	99	+6.9	+7.2	+6.2	+5.2	+1.9	−2	−5	−2	−16
1859	45	−1.8	1879	29	−9.3	−10.3	−3.3	−5.8	−3.4	−8	−7	−17	−4
1860	51	+0.7	1880	62	+4.8	+9.4	+5.2	+3.7	+2.4	−0	+3	+5	+4
1861	52	+0.6	1881	69	+6.0	−2.7	+10.7	−1.4	0	+2	+5	+10	+8
1862	48	−0.1	1882	44	−3.7	−5.7	+1.7	+0.4	+0.5	+7	−6	−8	+12
1863	54	+1.0	1883	30	−6.6	−5.6	−6.9	−1.6	−2.7	−6	+8	−5	+7
1864	54	+1.2	1884	44	−1.8	−4.0	+0.1	−0.1	+1.7	+14	−9	+2	−7
1865	54	+0.3	1885	35	−4.4	−4.6	−2.2	+0.7	−1.0	+11	+1	+11	+1
1866	55	+1.4	1886	20	−6.4	−7.1	−2.4	−1.9	+0.6	+5	−9	+13	−14
1867	48	−5.7	1887	25	−4.1	+2.3	−4.3	+2.6	+0.4	−5	−3	−12	+8
1868	39	−3.6	1888	22	−4.2	−4.4	−5.0	−1.2	−0.4	−1	−1	+1	−2
1869	47	−1.2	1889	6	−10.7	−1.5	−10.0	−1.2	−2.5	−1	+1	−5	−2
1870	49	−1.3	1890	54	+1.7	+2.1	−0.3	−0.5	+0.9	−9	−8	−15	−8
1871	47	+0.7	1891	70	+8.5	+2.4	+9.4	+3.0	+4.0	−5	0	0	+1
1872	46	+0.2	1892	50	−3.3	−3.7	−2.8	+0.1	−2.6	+11	+4	+16	−4
1873	44	−0.3	1893	35	−3.9	−6.1	−2.0	−2.0	−1.0	0	+11	+2	+16
1874	50	−0.1	1894	42	−3.8	−4.3	−2.3	−1.2	0	+6	+0	+11	−8
1875	56	+1.6	1895	58	+3.0	+3.3	+3.2	+3.3	+2.9	+3	+3	+5	+1
1876	35	−2.9	1896	38	−4.2	−4.1	−6.0	−1.2	−2.8	−6	−2	+3	0
1877	44	−1.1	1897	38	−1.6	−3.5	+0.1	0	+1.8	−8	−3	−13	−4
1878	74	+4.2	1898	29	−4.8	−6.0	−4.7	+0.4	−1.2	+4	+6	+14	+3
1879	46	−2.1	1899	21	−7.0	−7.8	−6.3	−2.2	−1.3	−1	−3	−2	−3
1880	58	+3.2	1900	35	−3.1	−1.6	−1.6	+0.5	−0.2	+3	−1	+8	+2
1881	56	+1.2	1901	76	+2.8	−1.4	−2.4	−2.8	−0.3	−1	−3	−7	−10
1882	54	+0.6	1902	68	+7.4	+16.0	+6.0	+5.6	+4.7	−3	−6	−11	−12
1883	34	−4.8	1903	39	−5.8	+4.7	−4.0	−0.5	−3.1	−3	+5	+4	+18
1884	45	−1.9	1904	55	+1.4	+0.7	+1.3	−1.6	−0.6	−2	+1	−7	−5
1885	31		1905	63	+1.6	−1.7	+3.3	−0.8	+1.7	+3	+14	−1	+34
1886	31		1906	69	+5.4	+6.1	−0.1	+3.3	+0.3	−6	−4	−1	−10
1887	31		1907	45	−3.0	−4.5	−2.0	−2.3	−2.0	+6	+4	+7	−10
1888	22		1908	45	−0.9	−4.3	+2.0	−0.1	+1.7	−12	+8	−10	+17
1889	20		1909	43	−1.6	−2.9	−3.3	+0.9	−0.5	+2	+10	+22	+14
1890	45		1910	25	−6.3	−4.0	−5.6	−1.0	−0.8	−6	−1	−6	−1
1891			1911	27	−2.3	−3.3	+0.0	+0.9	−0.6	−4	−4	−14	−8
1892			1912	3	−2.2	−10.0	−4.5	−5.6	−2.3	−1	+2	−2	+5
1893			1913	67	+9.8	+15.0	+6.6	+5.3	+4.6	−1	−3	−11	−14
1894			1914	65	+5.7	+7.1	+3.1	+1.0	−0.2	−6	−11	−5	−14
1899			1915	54	+0.8	+5.2	−0.7	−0.8	−2.4	+7	+8	+7	+21
1896			1916	65	+3.9	+0.4	+0.6	−0.3	+2.6	+4	−23	+14	−19
1897			1917	56	+0.8	−0.3	+0.5	+0.4	−1.1	+10	+3.3	0	+27
1898			1918	54	+0.3	+1.9	−0.5	−0.1	+0.2	−11	+4	−5	+15
1899			1919	53	−0.3	+0.6	−2.4	−0.1	−0.4	+6	−3	0	−5
1900			1920	34	−2.8	−0.3	−3.1	−0.8	−1.3	−2	−6	+10	−17
1901			1921	62	+1.1	−1.0	+3.0	−0.4	+2.1	−1	0	+5	+1
1902			1922	64	+3.2	+4.0	−3.6	+1.2	−1.8	+3	−4	+4	−10
1903			1923	60	+1.3	+3.3	−1.7			−5	−4	−2	−10
1904			1924	84	+14.5	+11.3	+13.1			−4	−3	−24	0
1905			1925	62	+5.6	+6.3	+3.0			−10	+11		

TABLE 2.—*The 28-month epochs in solar data and United States temperature and the intervals and lags between the epochs, actual and smoothed*

H. L. epochs				S. S. epochs				U. S. temp. epochs				Lags			
Max.	Min.	Intervals		Max.	Min.	Intervals		Min.	Max.	Intervals		H. L. to S. S.		H. L. to temp.	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1855.0				1857.7				1856.5				2.7		1.5	
	1856.7	2.7			1858.7	2.3	2.60		1858.2	3.0		2.0	2.33	1.7	1.67
1857.7		2.0	2.67	1860.0		2.8	2.51	1859.5		2.3	2.60	2.3	2.37	1.8	1.77
	1858.7	3.3	3.03		1861.5	2.5	2.50		1860.5	2.5	2.43	2.8	2.20	1.8	1.53
1861.0		3.8	3.37	1862.5		2.2	2.47	1862.0		2.5	2.40	1.5	1.83	1.0	1.10
	1862.5	3.0	3.10		1863.7	2.7	2.73		1863.0	2.2	2.40	1.2	1.30	.5	.57
1864.0		2.5	2.67	1865.2		3.3	3.17	1864.2		2.5	2.50	1.2	1.47	.2	.40
	1865.0	2.5	2.67		1867.0	3.5	3.17		1865.5	2.8	2.60	2.0	1.80	.5	.40
1866.5		3.0	2.67	1868.7		2.7	2.67	1867.0		2.5	2.60	2.2	1.97	.5	.33
	1868.0	2.5	2.50		1869.7	1.8	2.17		1868.0	2.5	2.67	1.7	1.80	.0	.33
1869.0		2.0	2.33	1870.5		2.0	2.00	1869.5		3.0	2.90	1.5	1.63	.5	.50
	1870.0	2.5	2.50		1871.7	2.2	2.07		1871.0	3.2	3.13	1.7	1.47	1.0	.73
1871.5		3.0	2.67	1872.7		2.0	2.07	1872.7		3.2	2.97	1.2	1.20	1.2	1.13
	1873.0	2.5	2.50		1873.7	2.0	2.10		1874.2	2.5	2.57	.7	.87	1.2	1.20
1874.0		2.0	2.07	1874.7		2.3	2.27	1875.2		2.0	2.10	.7	.80	1.2	1.20
	1875.0	1.7	1.90		1876.0	2.5	2.60		1876.2	1.8	1.87	1.0	1.07	1.2	1.23
1875.7		2.0	2.07	1877.2		3.0	2.83	1877.0		1.8	1.87	1.5	1.50	1.3	1.17
	1877.0	2.5	2.23		1879.0	3.0	2.67		1878.0	2.0	1.90	2.0	1.83	1.0	1.03
1878.2		2.2	2.23	1880.2		2.0	2.27	1879.0		2.0	2.00	2.0	1.93	.8	.87
	1879.2	2.0	2.07		1881.0	1.8	1.93		1880.0	2.0	2.00	1.8	1.87	.8	.80
1880.2		2.0	2.00	1882.0		2.0	1.93	1881.0		2.0	2.17	1.8	1.80	.8	.80
	1881.2	2.0	2.10		1883.0	2.0	1.90		1882.0	2.5	2.33	1.8	1.80	.8	.97
1882.2		2.3	2.27	1884.0		1.7	1.80	1883.5		2.5	2.33	1.8	1.60	1.3	1.03
	1883.5	2.5	2.43		1884.7	1.7	1.90		1884.5	2.0	2.23	1.2	1.33	1.0	1.03
1884.7		2.5	2.50	1885.7		2.3	2.27	1885.5		2.2	2.23	1.0	1.07	.8	.83
	1886.0	2.5	2.40		1887.0	2.8	2.77		1886.7	2.5	2.40	1.0	1.10	.7	.77
1887.2		2.2	2.17	1888.5		3.2	3.07	1888.0		2.5	2.40	1.3	1.43	.8	.83
	1888.2	1.8	1.83		1890.2	3.2	3.07		1889.2	2.2	2.23	2.0	2.00	1.0	1.00
1889.0		1.5	2.00	1891.7		2.8	2.67	1890.2		2.3	2.50	2.7	2.67	1.2	1.33
	1889.7	2.7	2.57		1893.0	2.0	2.27		1891.5	3.0	2.77	3.3	2.67	1.8	1.50
1891.7		3.5	3.17	1893.7		2.0	2.10	1893.2		3.0	3.10	2.0	2.37	1.5	1.53
	1893.2	3.5	3.50		1895.0	2.3	2.17		1894.5	3.3	3.27	1.8	1.53	1.3	1.37
1895.2		3.5	3.17	1896.0		2.2	2.23	1896.5		3.5	3.10	.8	1.03	1.3	1.30
	1896.7	2.5	2.83		1897.2	2.2	2.30		1898.0	2.5	2.73	.5	.60	1.3	1.30
1897.7		2.5	2.50	1898.2		2.5	2.40	1899.0		2.2	2.40	.5	.50	1.3	1.20
	1899.2	2.5	2.50		1899.7	2.5	2.67		1900.2	2.5	2.33	.5	.50	1.0	1.20
1900.2		2.5	2.43	1900.7		3.0	2.83	1901.5		2.3	2.43	.5	.60	1.3	1.03
	1901.7	2.3	2.37		1902.7	3.0	2.67		1902.5	2.5	2.77	1.0	.83	.8	1.20
1902.5		2.3	2.70	1903.7		2.0	2.60	1904.0		3.5	3.07	1.2	.90	1.5	1.43
	1904.0	3.5	3.17		1904.7	2.8	2.70		1906.0	3.2	3.07	.7	.80	2.0	1.57
1906.0		3.7	3.30	1906.5		3.3	2.93	1907.2		2.5	2.67	.5	.50	1.2	1.33
	1907.7	2.7	2.97		1908.0	2.7	3.00		1908.5	2.3	2.27	.3	.43	.8	.93
1908.7		2.5	2.73	1909.2		3.0	3.07	1909.5		2.0	2.27	.5	.53	.8	.63
	1910.2	3.0	2.60		1911.0	3.5	3.23		1910.5	2.5	2.50	.8	.77	.3	.47
1911.7		2.3	2.43	1912.7		3.2	3.17	1912.0		3.0	2.67	1.0	1.17	.3	.53
	1912.5	2.0	2.43		1914.2	2.8	2.83		1913.5	2.5	2.67	1.7	1.50	1.0	.70
1913.7		3.0	2.67	1915.5		2.5	2.43	1914.5		2.5	2.59	1.8	1.57	.8	.77
	1915.5	3.0	2.73		1916.7	2.0	2.00		1916.0	2.7	2.73	1.2	1.27	.5	.60
1916.7		2.2	2.57	1917.5		1.5	1.90	1917.2		3.0	2.83	.8	.83	.5	.77
	1917.7	2.5	2.50		1918.2	2.2	2.07		1919.0	2.8	2.67	.5	.60	1.3	.87
1919.2		2.8	2.70	1919.7		2.5	2.40	1920.0		2.2	2.50	.5	.40	.8	.93
	1920.5	2.8	2.60		1920.7	2.5	2.60		1921.2	2.5	2.40	.2	.30	.7	.67
1922.0		2.2	2.50	1922.2		3.0	3.00	1922.5		2.5	2.40	.2	.43	.5	.73
	1922.7	2.5			1923.7	3.5			1923.7	2.2		1.0	.80	1.0	.57
1924.5				1925.7				1924.7				1.2		.2	

TABLE 3.—*Epochs of the 11-year variation in the lengths of the 28-month period and in the lags*

Wolfer's epochs		Length of 28-month period						Lag			
		H. L.		S. S.		U. S. temp.		H. L. to S. S.		H. L. to U. S. temp.	
Max.	Min.	Long	Short	Long	Short	Long	Short	Long	Short	Long	Short
1860.1		1860.5			1861.0		1862.5		1863.2		1866.0
	1867.2		1868.5	1866.0		1871.0		1868.0		1874.0	
1870.6		1871.5			1871.0		1876.7		1873.7		1879.5
	1878.9		1877.5	1877.5		1882.5		1879.5		1883.5	
1883.9		1884.5			1883.0		1889.0		1885.7		1887.0
	1889.6		1888.5	1889.2		1894.5		1891.2		1891.5	
1894.1		1893.2			1894.2		1900.0		1898.0		1900.0
	1901.7		1900.5	1901.0		1905.0		1903.0		1905.0	
1906.4		1905.2			1904.0		1909.0		1908.0		1911.0
	1913.6		1912.0	1911.5		1917.0		1914.0		1919.2	
1917.6		1916.0			1917.5		1923.0		1920.5		1924.5
	1923.6		1922.0	1923.7							

TABLE 4.—*New epochs of 11-year variation in relative numbers and intervals between epochs*

Maxima	Minima	Intervals		Maxima	Minima	Intervals		Epochs of magnetic declination range	
		(1)	(2)			(1)	(2)	Maxima	Minima
1750.3				1837.3		9.8	9.6	1837.7	
	1755.5	11.0	11.2		1843.6	11.2	10.9		1843.8
1761.3		10.7	11.3	1848.5		12.5	12.5	1848.8	
	1766.2	8.7	8.2		1856.1	11.8	12.0		1856.3
1770.0		9.3	9.0	1860.3		10.9	11.2	1859.9	
	1775.5	8.6	8.7		1867.0	10.7	10.5		1867.0
1778.6		8.9	9.2	1871.0		11.6	11.7	1871.2	
	1784.4	9.7	9.7		1878.6	12.7	13.3		1878.5
1788.3		14.2	13.6	1883.7		10.7	10.7	1884.0	
	1798.6	15.8	17.1		1889.3	10.1	10.2		1889.5
1804.1		12.1	12.3	1893.8		12.3	12.1	1893.6	
	1810.7	12.7	11.2		1901.6	12.8	12.3		1901.8
1816.8		12.3	12.7	1906.6		11.3	11.9	1906.5	
	1823.0	12.9	13.5		1912.9	11.3	11.2		1913.0
1829.7		10.8	10.6	1917.9		10.5	10.0	1917.7	
	1833.8	7.6	7.3		1923.4				1923.7

[1] Intervals from new epochs. [2] Intervals from old epochs.

TABLE 5.—*Sun spot and magnetic epochs, 28-month period*

Sun spot		Magnetic		Sun spot		Magnetic	
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
1837.0		1836.5		1880.2		1879.2	
	1838.2		1837.5		1881.0		1880.2
1839.5		1838.7		1882.0		1881.7	
	1840.5		1839.7		1883.0		1882.7
1841.2		1841.0		1884.0		1884.0	
	1842.0		1842.2		1884.7		1885.0
1842.7		1843.2		1885.7		1886.7	
	1844.0		1844.2		1887.0		1887.0
1845.2		1845.2		1888.5		1888.0	
	1845.7		1845.7		1890.2		1889.5
1846.7		1846.7		1891.7		1891.2	
	1847.2		1847.2		1893.0		1892.5
1848.5		1848.7		1893.7		1893.5	
	1849.5		1849.5		1895.0		1894.5
1850.0		1850.5		1896.0		1895.7	
	1851.5		1851.5		1897.2		1897.7
1852.2		1852.5		1898.2		1898.7	
	1853.7		1853.5		1899.7		1900.5
1854.7		1854.7		1900.7		1901.5	
	1856.2		1856.2		1902.7		1902.5
1857.7		1857.5		1903.7		1904.7	
	1858.7		1858.7		1904.7		1906.5
1860.0		1860.0		1906.5		1907.5	
	1861.5		1861.5		1908.0		1909.5
1862.5		1863.0		1909.2		1910.2	
	1863.7		1864.2		1911.0		1912.0
1865.2		1865.5		1912.7		1913.2	
	1867.0		1867.2		1914.2		1914.2
1868.7		1868.7		1915.5		1915.7	
	1869.7		1869.7		1916.7		1916.5
1870.5		1870.7		1917.5		1917.7	
	1871.7		1871.7		1918.2		1919.2
1872.7		1872.7		1919.7		1920.0	
	1873.7		1873.5		1920.7		1921.5
1874.7		1874.5		1922.2		1922.7	
	1876.0		1875.5		1923.7		1924.0
1877.2		1876.7		1925.7		1925.7	
	1879.0		1878.0				

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THE RATE OF DECAY OF ATMOSPHERIC EDDIES

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INTRODUCTION

The way in which eddies are produced and the rate at which their energy is dissipated in actual fluids is a matter of considerable importance, both from a practical standpoint as a measure of the effect of viscosity on atmospheric motions in its relation to aerodynamical and meteorological problems, and also for theoretical reasons as a means of furnishing a desirable basis for, and a check upon the mathematical investigations regarding the motion of perfect fluids; that is, fluids in which similar motions may occur and which may have all of the other properties of actual fluids, but which we may imagine to be entirely devoid of internal friction, or viscosity.

In the present paper it is attempted to investigate the dynamical nature of individual eddies, their growth and rate of decay, etc., rather than to make a statistical study of the combined behavior and average effect of a large number of them, which has already been so admirably done in the work of a number of other students of this subject.

The theory is more or less restricted to eddy motions taking place at a fixed altitude and with supposedly the least possible exchange of thermal and gravitational energy, and therefore only a slight reference will be found to that part of the problem relating to the motion of eddies which may, over a given time, be increasing, or decreasing with height relative to any fixed levels.

OBSERVATIONS ON THE DISSIPATION OF EDDY MOTION

Selecting certain days when the air was very clear and quiet, some attempts were made to determine the rate of decay of eddies set up and following in the rear of railway trains, motor cars, and other vehicles moving on, or near, the earth's surface.

The methods employed in this investigation were necessarily of such nature as to afford but little opportunity for making any very exact measurements in the process of collecting the required data, since there were no means available for making an accurate quantitative study of the velocities and time intervals involved, as is otherwise the case where the experiments are under the complete control of the observer, and therefore the results obtained in the present work can hardly be considered as being much more than rough approximations made in the absence of any better and more exact facilities for experiment.

The usual procedure in this investigation was to stand as near the track as possible until the train had passed by and then to release bits of paper or other light material, and observe the lapse of time and how far the train traveled before the eddies caused by its motion had become entirely dissipated.

The distance which the train moved during the interval could not be very accurately estimated in most instances, but knowing the usual speed of the particular train passing at that time and by taking note of certain objects along the right of way which were apparently about even with the rear of the train by the time the eddies, as indicated by the motion of the bits of paper, had completely died away; then by measuring this distance it was possible to obtain a rough estimate of how far the train had traveled during that period of time.

It was found that in the case of a fast passenger train moving at a speed of around 25 meters per second (or about 56 miles per hour), the eddies formed by its motion